SAK ACADEMY

XI,XII,NEET/AIIMS,IIT-JEE (MAIN+ADV.)

TOPICS:-

1.ADDITION OF VECTOR
2.HEAD TO TAIL METHOD OF ADDITION OF VECTOR
3.TRIANGULAR LAW OF VECTOR ADDITION
4.PARALLELOGRAM LAW OF VECTOR ADDITION
5.POLYGON LAW OF VECTOR ADDITION
6.SPECIAL CASES OF VECTOR ADDITION
7.SUBTRACTION OF VECTOR
8.SPECIAL CASES OF SUBTRACTION OF VECTOR
9.NUMERICALS BASED ON ADDITION & SUBTRACTION OF VECTOR

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Introduction:- Vectors cannot be added by simple laws of algebra, which are applicable to scalers. To add two or more vectors, we must follow certain laws which are known as laws of vector addition.

These are as follow:-

Method-I :- Tip to tail method / law
Method-II :- Triangle law of vector addition
Method-III :- Parallelogram law of vector addition
Method-IV :- Polygon law of vector addition

Tip-to-tail or Head-to-tail method:-
The easiest way to add vectors is the tip-to-tail or head-to-tail method. Remember that the only two important things about vectors are length and direction. Therefore we can move any vector to any location in the plane as we like and, as long as we do not change the length and direction, it remains the same vector in same plane.

Adding by the tip-to tail method means to move one vector so that its tail lies on the tip of the first vector. The resultant vector ($\vec{R} = \vec{A} + \vec{B}$) is the sum of the two vectors is simply the new vector drawn from the origin of the first vector to the arrow of the second vector.

For example, Let two or more vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ are give shown in fig.

Magnitude of the resultant vector

Let the two vectors $\vec{A}$ and $\vec{B}$, inclined at an angle θ be acting on a particle at same time. Let they be represented in magnitude and direction by two sides $\vec{OP}$ and $\vec{PQ}$ of triangle OPQ, taken in the same order. Then according to triangle law of vectors addition, the resultant $\vec{R}$ is represented by the third side $\vec{OQ}$ of triangle, taken in opposite order.

Now In ΔPQN, $\cos\theta = \frac{PN}{B} \Rightarrow PN = B\cos\theta$

and $\sin\theta = \frac{QN}{B} \Rightarrow QN = B\sin\theta$

Now. In ΔONQ, By pythagoras theorem, we get

$OQ^2 = ON^2 + QN^2 \Rightarrow OQ^2 = (OP+PN)^2 + QN^2$

$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

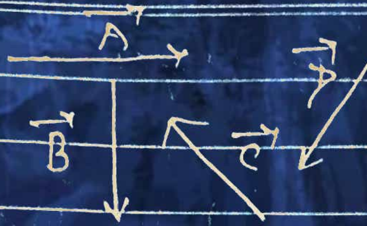
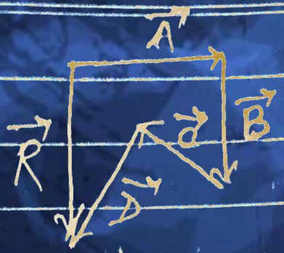
$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \text{ OR } R = \left(A^2+B^2+2AB\cos\theta\right)^{1/2}$$

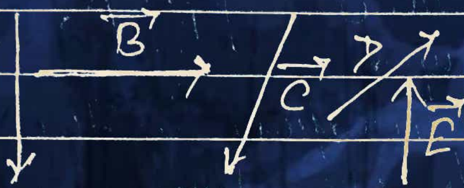
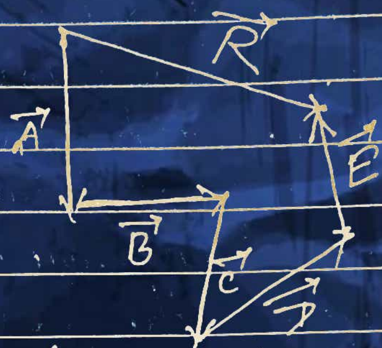
Direction of resultant vectors:

If θ is angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$ but

(i)  then 

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

(ii)  

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

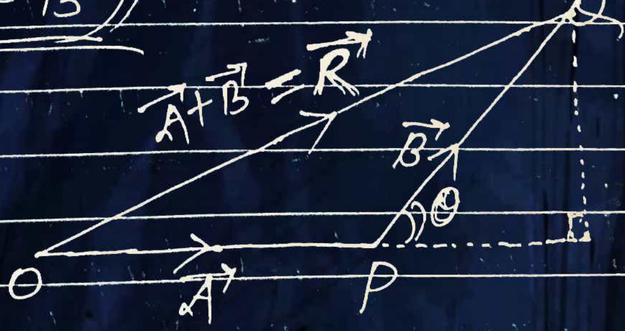
⁎ Triangle law of vector addition of two vectors :→

It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

i.e. $\boxed{\vec{R} = \vec{A} + \vec{B}}$

$$\because \vec{OQ} = \vec{OP} + \vec{PQ}$$

$$\therefore \boxed{\vec{R} = \vec{A} + \vec{B}}$$



Note: This is the geometrical method of vector addition.

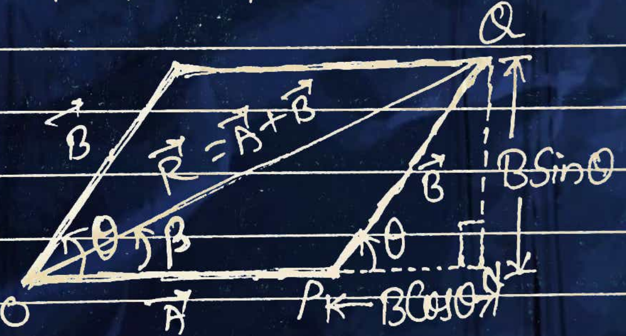
If $\vec{R}$ makes an angle α with $\vec{A}$ then in ΔOQN, $\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP+PN}$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}} \text{ OR } \boxed{\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)}$$

Parallelogram law of vector addition :→

If two non-zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

Magnitude of the resultant vector



Since, $R^2 = ON^2 + QN^2$

$\Rightarrow R^2 = (OP+PN)^2 + QN^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\therefore R = |\vec{R}| = |\vec{A} + \vec{B}|$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Direction of resultant vector :-

$$\boxed{\tan\beta = \frac{QN}{ON} = \frac{B\sin\theta}{A+B\cos\theta}} \text{ OR } \boxed{\beta = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)}$$

Special cases :-

Case-I When two vectors are acting in the same direction then $\theta = 0°$ and $\boxed{R = A+B}$

$\tan\beta = \frac{B \times 0}{A + B(1)} = 0$ or $\boxed{\beta = 0°}$

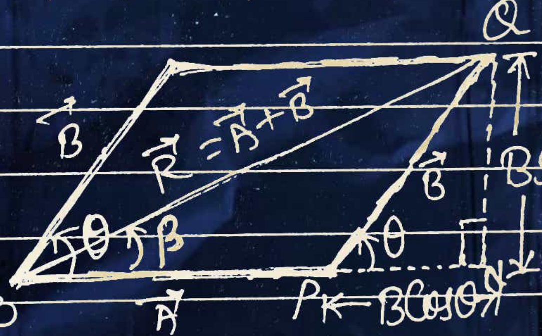
If $\vec{R}$ makes an angle α with $\vec{A}$ then in $\triangle OQN$, $\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP+PN}$

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} \quad \text{OR} \quad \alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

Parallelogram law of vector addition:→

If two non-zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

Magnitude of the resultant vector



Since, $R^2 = ON^2 + QN^2$

$\Rightarrow R^2 = (OP+PN)^2 + QN^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\therefore R = |\vec{R}| = |\vec{A}+\vec{B}|$

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$

Direction of resultant vector:-

$$\tan\beta = \frac{QN}{ON} = \frac{B\sin\theta}{A+B\cos\theta} \quad \text{OR} \quad \beta = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

Special cases:-

Case-I When two vectors are acting in the same direction then $\theta = 0°$ and $R = A+B$

$$\tan\beta = \frac{B \times 0}{A+B(1)} = 0 \text{ or } \beta = 0°$$

Note:- Thus, for two vectors acting in the same direction, the magnitude of the resultant vector is equal to the sum of the magnitude of the two vectors and act along the direction of $\vec{A}$ and $\vec{B}$.

Case-II When two vectors are acting in opposite directions, then $\theta = 180°$ and

$$R = \sqrt{A^2 + B^2 + 2AB(-1)} = \sqrt{(A-B)^2} = A-B$$

Hence, $R = |A-B|$ or $|B-A|$ when $\theta = 180°$

For direction,

$$\tan\beta = \frac{B \times 0}{A + B(-1)} = 0 \text{ or } \beta = 0° \text{ or } 180°$$

Note:- Thus, for two vectors acting in opposite directions, the magnitude of the resultant vector is equal to the difference of the magnitudes of the two vectors and acts in the direction of bigger vector.

Case-III When two vectors are acting perpendicular to each-other then $\theta = 90$ and $R = \sqrt{A^2 + B^2}$ but

For direction

$$\tan\beta = \frac{B \sin 90°}{A + B\cos 90} \Rightarrow \tan\beta = \frac{B}{A} \Rightarrow \beta = \tan^{-1} B/A$$

Case-IV Resultant of two vectors will be maximum when they are parallel, i.e. angle between them is 0° or $R_{max} = A+B$

Case-V Resultant of two vectors will be minimum when they are anti-parallel, i.e. angle between them is 180°.
or, $R_{min} = A - B$

Case-VI – Combining Case-IV & Case-V, We can conclude that the range of the Resultant vector will be

$$R_{min} \le R \le R_{max} \Rightarrow |\vec{A}-\vec{B}| \le R \le |\vec{A}+\vec{B}|$$

Case-VII Resultant of two vectors of unequal magnitude can never be zero.

Case-VIII :- Direction of resultant vector
Let θ be the angle between $\vec{A}$ and $\vec{B}$
then $|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$ but
If R makes an angle α or β with $\vec{A}$ or $\vec{B}$,
then $\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta}$ & $\tan\beta = \dfrac{A\sin\theta}{B+A\cos\theta}$

Case-IX If $|\vec{A}| = |\vec{B}| = F$ then
$|\vec{A}+\vec{B}| = R = \sqrt{A^2+B^2+2AB\cos\theta}$
$|\vec{A}+\vec{B}| = R = \sqrt{F^2+F^2+2\cdot F\cdot F\cos\theta}$
$R = \sqrt{2F^2+2F^2\cos\theta}$
$R = \sqrt{2F^2(1+\cos\theta)}$
$R = \sqrt{2F^2\cdot 2\cos^2\theta/2}$ $[\because 1+\cos\theta = 2\cos^2\theta/2]$

$$R = 2F\cos\theta/2$$

Case-X If $|\vec{A}| = |\vec{B}| = F$ then

$$|\vec{A} - \vec{B}| = R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$R = \sqrt{F^2 + F^2 - 2 \cdot F \cdot F\cos\theta}$$

$$R = \sqrt{2F^2 - 2F^2\cos\theta}$$

$$R = \sqrt{2F^2(1 - \cos\theta)}$$

$$R = \sqrt{2F^2 \cdot 2\sin^2\theta/2}$$

$$\boxed{R = 2F\sin\theta/2}$$

Q. 01. Two forces whose magnitude are in the ratio 3:5 give a resultant of 28N. If the angle of their inclination is 60°, find the magnitude of each force.

Sol^n:- Let F_1 and F_2 be the two forces. and let the ratio quantity be x.

Then $F_1 = 3x$ and $F_2 = 5x$

Since $R = 28N$ and $\theta = 60°$

$$\therefore \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = R$$

$$\sqrt{(3x)^2 + (5x)^2 + 2 \cdot 3x \cdot 5x \cdot \cos 60} = 28$$

$$\sqrt{9x^2 + 25x^2 + 2 \cdot 3x \cdot 5x \cdot \frac{1}{2}} = 28$$

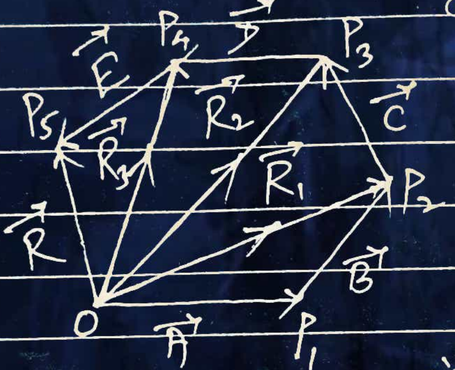
$$\sqrt{49x^2} = 28$$

$$7x = 28 \Rightarrow x = 28/7 = 4$$

When $x = 4$ then $F_1 = 3x = 3 \times 4 = 12N$ &

$F_2 = 5x = 5 \times 4 = 20N$

Polygon law of vector addition for more than two vectors:- It states that if a number of non zero vectors are represented by the (n-1) sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order.



Since, $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ and $\vec{E}$ are represented by sides of an open polygon $OP_1P_2P_3P_4$ in same order and its resultant vector $\vec{R}$ is represent by the side OP_5 in opposite order of this closed polygon.

So, $\boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}}$

Proof By using tringular law of vector addition

In ΔOP_1P_2 $\quad \vec{R_1} = \vec{A} + \vec{B}$ —— ①

Now, In ΔOP_2P_3, $\vec{R_2} = \vec{R_1} + \vec{C} = \vec{A} + \vec{B} + \vec{C}$
$[\because \vec{R_1} = \vec{A} + \vec{B}]$

Again, In ΔOP_3P_4, $\vec{R_3} = \vec{R_2} + \vec{D}$
$\vec{R_3} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$ $[\because \vec{R_2} = \vec{A} + \vec{B} + \vec{C}]$

Thus, In ΔOP_4P_5, $\vec{R} = \vec{R_3} + \vec{E}$
$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$ $[\because \vec{R_3} = \vec{A} + \vec{B} + \vec{C} + \vec{D}]$

Note:- (i) Resultant of two unequal vectors cannot be zero.
(ii) Resultant of three co-planar vectors may

or may not be zero.

(iii) Resultant of three non-co-planar vectors can not be zero.

(iv) Resultant of two vectors is always located in their common plane.

(v) Vector addition is commutative. i.e. $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

(vi) Vector addition is associative i.e. $\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$.

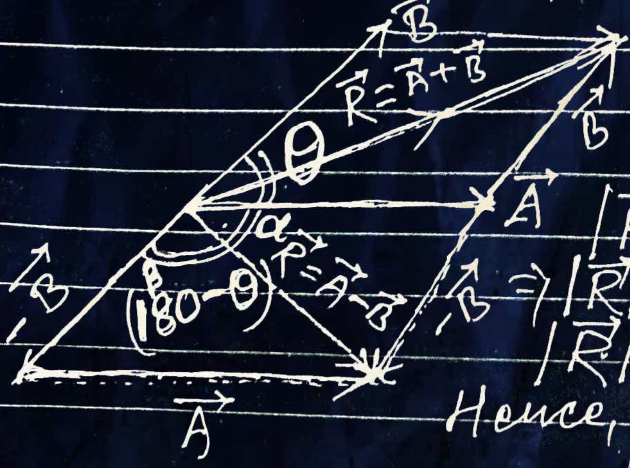
(vii) If vectors are of unequal magnitude then minimum three coplanar vectors are required for zero resultant.

Subtraction of two vectors :- Since negative of a vector say $-\vec{A}$ is a vector of the same magnitude as vector $\vec{A}$ but pointing in a direction opposite to that of $\vec{A}$.

Thus, $\vec{A}-\vec{B}$ can be written as $\vec{A}+(-\vec{B})$ or $\vec{A}-\vec{B}$ or subtraction of two vectors is really the vector addition of $\vec{A}$ and $-\vec{B}$

i.e.

Let angle between two vectors $\vec{A}$ and $\vec{B}$ is θ. Then angle between $\vec{A}$ and $-\vec{B}$ will be $(180-\theta)$ as shown in the figure.



Magnitude of resultant vector $|\vec{R}|=|\vec{A}-\vec{B}|$ will be

$$|\vec{R}|=|\vec{A}-\vec{B}|=\sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$\Rightarrow |\vec{R}|=\sqrt{A^2+B^2+2AB(-\cos\theta)}$$

$$|\vec{R}|=\sqrt{A^2+B^2-2AB\cos\theta}$$

Hence, $\boxed{R=\sqrt{A^2+B^2-2AB\cos\theta}}$

For direction of resultant vector $\vec{R}$ will either calculate angle α or β where

$$\tan\alpha = \frac{B\sin(180-\theta)}{A+B\cos(180-\theta)} = \frac{B\sin\theta}{A-B\cos\theta}$$

OR

$$\tan\beta = \frac{A\sin(180-\theta)}{B+A\cos(180-\theta)} = \frac{A\sin\theta}{B-A\cos\theta}$$

Special cases:-

Case-I When $\theta = 0°$

then $R = |\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos 0°}$

$\vec{R} = \sqrt{(A-B)^2} = A-B$

and $\tan\alpha = \frac{B\sin 0°}{A-B\cos 0°} = 0 \Rightarrow \alpha = 0°$ OR $\beta = 0°$

Case-II When $\theta = 180°$

then $R = \sqrt{A^2+B^2-2AB\cos 180°}$

$|\vec{A}-\vec{B}| = R = \sqrt{A^2+B^2-2AB(-1)} = A+B$

and $\tan\alpha = \frac{B\sin 180°}{A-B\cos 180} = 0 \Rightarrow \alpha = 0°$ & $\beta = 0°$

Case-III When $\theta = 90°$ then $R = \sqrt{A^2+B^2}$

and $\tan\alpha = \frac{B\sin 90}{A-B\cos 90} = \frac{B}{A} \Rightarrow \alpha = \tan^{-1}(B/A)$ &

Similarly $\beta = \tan^{-1}(A/B)$.

Case-IV If $\vec{A}+\vec{B} = \vec{A}-\vec{B}$ then $\vec{B} = 0$ (a null vector)

Case-V If two vectors are such that their sum and their difference have equal magnitude, then angle between the given vectors $\theta = 90°$.

Proof :- $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

Squaring on both sides, we get

$$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$$

$$4AB\cos\theta = 0 \Rightarrow \cos\theta = 0° \Rightarrow \theta = 90°$$

Case-VI :- If two vectors have equal magnitude Then $\boxed{R = 2F\sin\theta/2}$

Proof Let $|\vec{A}| = |\vec{B}| = F$ $\because R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

Then $|\vec{A} - \vec{B}| = R = \sqrt{F^2 + F^2 - 2 \cdot F \cdot F\cos\theta}$

$$R = \sqrt{2F^2 - 2F^2\cos\theta} = \sqrt{2F^2(1 - \cos\theta)}$$

$$R = \sqrt{2F^2 \cdot 2\sin^2\theta/2} \quad [\because 2\sin^2\theta/2 = 1 - \cos\theta]$$

Thus, $\boxed{R = 2F\sin\theta/2}$

Case-VII If $\theta = 60°$ then $\boxed{R = 2F\sin\theta/2 = F}$

i.e. $|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}| = F$ at $\theta = 60°$

Case-VII (i) The vector subtraction does not follow commutative law. i.e. $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

(ii) The vector subtraction does not follow associative law. i.e. $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

Q<01> $\vec{A}+\vec{B}$ can also be written as
<A> $\vec{A}-\vec{B}$ <B> $\vec{B}-\vec{A}$ <C> $\vec{B}+\vec{A}$ <D> $\vec{B}\cdot\vec{A}$

Answer (C)

Q<02> If $\vec{P}+\vec{Q}=0$, then which of the following is necessarily true?
<A> $\vec{P}=0$ <B> $\vec{P}=-\vec{Q}$ <C> $Q=0$ <D> $\vec{P}=\vec{Q}$

Answer (B)

Q<03> Two vectors having magnitudes 8 and 10 can have maximum and minimum value of magnitude of their resultant as
<A> 12, 6 <B> 10, 3 <C> 18, 2 <D> None of these

Answer (C)

Q<04> $(\vec{P}+\vec{Q})$ is a unit vector along x-axis. If $\vec{P}=\hat{i}-\hat{j}+\hat{k}$, then what value does Q have
<A> $\hat{i}+\hat{j}-\hat{k}$ <B> $\hat{j}-\hat{k}$ <C> $\hat{i}+\hat{j}+\hat{k}$ <D> $\hat{j}+\hat{k}$

Answer (B)

Q<05> For the resultant of two vectors to be maximum what must be the angle between them
<A> 0° <B> 60° <C> 90° <D> 180°

Answer (A)

Q<06> The minimum number of forces of unequal magnitude whose vector sum can equal to zero is

<A> Two <B> Three <C> Four <D> Any

Answer (B)

Q <07> Given that $\vec{P}+\vec{Q}+\vec{R}=0$. Which of the following statement is true?
<A> $|\vec{P}|+|\vec{Q}|=|\vec{R}|$ <B> $|\vec{P}+\vec{Q}|=|\vec{R}|$ <C> $|\vec{P}|-|\vec{Q}|=|\vec{R}|$
<D> $|\vec{P}-\vec{Q}|=|\vec{R}|$

Answer (B)

Q. <08> If $\vec{A}=2\hat{i}+\hat{j}$, $\vec{B}=3\hat{i}-\hat{k}$ and $\vec{C}=6\hat{i}-2\hat{k}$, then the value of $\vec{A}-2\vec{B}+3\vec{C}$ would be

Answer (B)

Q (09.) If $\vec{A}=4\hat{i}-3\hat{j}$ and $\vec{B}=6\hat{i}+8\hat{j}$, then magnitude and direction of $\vec{A}+\vec{B}$ with X-axis will be. <A> $5, \tan^{-1}(3/4)$
<B> $5\sqrt{5}, \tan^{-1}(1/2)$ <C> $10, \tan^{-1}(5)$ <D> $25, \tan^{-1}(3/4)$

Answer <B>

Q (10) A truck travelling due north at $20\,ms^{-1}$ turns west and travels at the same speed. The change in its velocity will be <A> $40\,ms^{-1}$ N-W <B> $20\sqrt{2}\,ms^{-1}$ N-W
<C> $40\,ms^{-1}$ S-W <D> $20\sqrt{2}\,ms^{-1}$ S-W.

Answer (D)

Q (11) The resultant of two vectors $\vec{A}$ and $\vec{B}$ is given by $|\vec{R}|=\{|\vec{A}|-|\vec{B}|\}$, the angle between $\vec{A}$ and $\vec{B}$ will be
<A> 90° <B> 180° <C> 0° <D> None of these

Answer (B)

Q<12> If $|\vec{A}| = 2$ and $|\vec{B}| = 4$ and angle between them is 60°, then $|\vec{A} - \vec{B}|$ is
<A> $\sqrt{13}$ <B> $3\sqrt{3}$ <C> $\sqrt{3}$ <D> $2\sqrt{3}$

Answer (D)

Q<13> If $\vec{A}$ and $\vec{B}$ are two vectors such that $|\vec{A} + \vec{B}| = 2|\vec{A} - \vec{B}|$, then the angle between vectors $\vec{A}$ and $\vec{B}$ is
<A> 45° <B> 60° <C> 30° <D> data insufficient

Answer (D)

Q(14) If $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$ then
(A) $\vec{P} = 0$ <B> $\vec{Q} = 0$ (C) $\vec{P} = 1$ (D) $|\vec{Q}| = 1$

Answer (B)

Q(15) The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector will be (A) $3\hat{i} + 5\hat{j} + 2\hat{k}$ <B> $3\hat{i} + 2\hat{j} + 5\hat{k}$ <C> $5\hat{i} + 3\hat{j} + 2\hat{k}$ <D> None of these.

Answer (B)

Q<16> If a particle moves from point P(2, 3, 5) to point Q(3, 4, 5). Its displacement vector is given by
<A> $\hat{i} + \hat{j} + 10\hat{k}$ <B> $\hat{i} + \hat{j} + 5\hat{k}$ <C> $\hat{i} + \hat{j}$
<D> $2\hat{i} + 4\hat{j} + 6\hat{k}$

Answer (C)

Q(17) At what angle should the two forces 2P and $\sqrt{2}P$ act so that the resultant force is $P\sqrt{10}$

<A> 45° <B> 60° <C> 90° <D> 120°

Answer (A)

Q<18> What vector must be added to the sum of two vectors $2\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}-2\hat{k}$ so that the resultant is a unit vector along z-axis?

<A> $5\hat{i}+\hat{k}$ <B> $-5\hat{i}+3\hat{j}$ <C> $3\hat{j}+5\hat{k}$ <D> $-3\hat{j}+2\hat{k}$

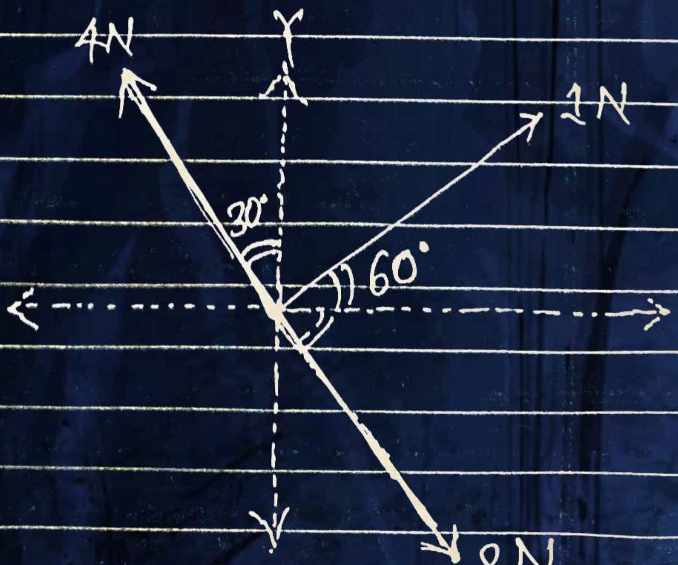
Answer (B)

Q(19) Three vectors each of magnitude A, are acting at a point such that angle between any two consecutive vectors in same plane is 60°. The magnitude of their resultant is

<A> 2A <B> $\sqrt{3}A$ <C> $\sqrt{2}A$ <D> $\sqrt{6}A$

Answer (A)

Q(20) Three forces acting on a body are shown in the figure. To have the resultant force only along the y-direction, the magnitude of the minimum additional force needed is



<A> $\frac{\sqrt{3}}{4}$N <B> $\sqrt{3}$N

<C> 0.5N <D> 1.5N

Q (21) Find the angle that the vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ makes with Y-axis.

<A> $\cos^{-1}(3)$ <B> $\cos(\sqrt{14})$ <C> $\cos^{-1}(3/\sqrt{14})$ <D> $\cos^{-1}\left(\frac{\sqrt{14}}{3}\right)$

Answer (C)

Q (22) If the mid-points of the consecutive sides of any quadrilateral are connected by straight line segments, prove that the resulting quadrilateral is a parallelogram.

Q (23) A particle is moving from point A(1, 2, 3) to the point B(4, 6, 9). Find its displacement vector.

<A> $4\hat{i} + 3\hat{j} + 6\hat{k}$ <B> $3\hat{i} + 4\hat{j} + 6\hat{k}$ <C> $4\hat{j} + 3\hat{i}$ <D> $6\hat{i}$

Answer (B)

Q (24) A vector is given by $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. Find the magnitude of $\vec{A}$, unit vector along $\vec{A}$ and angles made by $\vec{A}$ with coordinate axes.

Answer, $5\sqrt{2}$, $\frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$

$\cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$, $\cos^{-1}(4/5\sqrt{2})$, $\pi/4$

Q (25) If $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$, find the angle between $\vec{P}$ and $\vec{Q}$.

<A> π <B> $\pi/2$ (C) 2π <C> $2\pi/3$

Answer (B)

Q (26) The two equal vectors have a resultant equal to either of the two. Find the angle between them. <A> 60° <B> 120° <C> 180° <D> 90°

Answer 120°

Q (26) Two forces whose magnitude are in the ratio 3:5 give a resultant of 28 N. If the angle of their inclination is 60°, find the magnitude of each force.

<A> 12N, 20N <B> 20N, 40N <C> 3N, 5N <D> 30N, 10N

Answer (A)

Q (27) A boy walks 4m east and then 3m south. Find the displacement of the boy.

Answer 5m at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ south of east

Q (28) If $\vec{A} = \vec{B} + \vec{C}$ have scalar magnitudes of 5, 4, 3 units respectively, then find the angle between $\vec{A}$ and $\vec{C}$.

Ans: $\cos^{-1}(3/5)$

Q (29) The initial and final position vectors for a particle are $(-3m)\hat{i} + (2m)\hat{j} + (8m)\hat{k}$ and $(9m)\hat{i} + (2m)\hat{j} + (-8m)\hat{k}$ respectively, find the displacement of the particle.

Ans: $(12m)\hat{i} + (-16m)\hat{k}$

Q (30) One of the two rectangular components of a force is 20N and it makes an angle of 30° with the force. The magnitude of the other component is

<A> $10/\sqrt{3}$ <B> $20/\sqrt{3}$ <C> $15/\sqrt{3}$ <D> $40/\sqrt{3}$

Ans: $20/\sqrt{3}$

Q(31) Two vectors acting through a point are in the ratio 3:5. If the angle betweeen them is 60° and the magnitude of their resultant is 35. Find the magnitude of vectors.

(A) 10, 15 <B> 15, 25 <C> 15, 20 <D> 25, 30

Answer (B)

Q(32) Two forces acting in opposite direction have resultant 10N and when acting ~~perdind~~ perpendicularly have resultant 50N. Find the magnitude of forces are.

<A> 40, 30 <B> 10, 20 <C> 20, 30 <D> 50, 40

Answer <A>

Q(33) The sum of the magnitude of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude. what are the magnitude of forces?

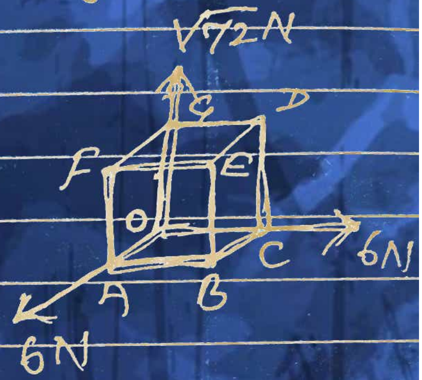
<A> 13, 18 <B> 13, 5 <C> 13, 15 <D> 5, 15

Answer (B)

Q(34) The resultant of two forces 3p and 2p is R. If the first force is doubled then resultant is also doubled. the angle between the two forces is <A> 60° <B> 120° <C> 30° <D> 45°

Answer 120°

Q (35) Three forces of magnitudes 6 N, 6 N and $\sqrt{72}$ N act at corner of cube along three sides as shown in figure. Resultant of these forces is



<A> 12N along OB <B> 18N along OA
<C> 18N along OC <D> 12N along OE

Answer (D)

Q (36) Resultant of two vectors $\vec{A}$ and $\vec{B}$ is of magnitude P. If $\vec{B}$ is reversed then resultant is of magnitude Q. What is the value of P^2+Q^2?

<A> $2(A^2+B^2)$ <B> $2(A^2-B^2)$ <C> A^2-B^2 <D> A^2+B^2

Answer (A)

Q (37) Vectors $\vec{A}$ and $\vec{B}$ include an angle θ between them. If $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ respectively subtend angles α and β with $\vec{A}$, then $(\tan\alpha + \tan\beta)$ is

<A> $\frac{AB\sin\theta}{(A^2+B^2\cos^2\theta)}$ <B> $\frac{2AB\sin\theta}{A^2-B^2\cos^2\theta}$ <C> $\frac{A^2\sin^2\theta}{A^2+B^2\cos^2\theta}$

<D> $\frac{B^2\sin^2\theta}{A^2-B^2\cos^2\theta}$

Answer (B)

Q (38) If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ will be

<A> 30° <B> 45° <C> 60° <D> 90°

Answer (D)

Q(39) Five equal forces of 10N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant forces will be
(A) zero (B) 10N (C) 20N (D) $10\sqrt{2}$N.

Answer (A)

Q(40) There are two forces, one of 5N. and other of 12N at what angle the two vectors be added to get resultant vectors of 17N, 7N and 13N respectively. (A) 0°, 180° & 90° (B) 0°, 90° & 180° (C) 0°, 90° & 90° (D) 180°, 0° & 90°

Answer (A)

Q(41) Forces F_1 and F_2 act on a point mass in two mutually perpendicular directions. The resultant force on the point mass will be
(A) $F_1 + F_2$ (B) $F_1 - F_2$ (C) $\sqrt{F_1^2+F_2^2}$ (D) $F_1^2+F_2^2$

Answer (C)

Q(42) Let the angle between two non-zero vectors $\vec{A}$ and $\vec{B}$ be 120° and resultant be $\vec{C}$
(A) $\vec{C}$ must be equal to $|\vec{A}-\vec{B}|$
(B) $\vec{C}$ must be less than $|\vec{A}-\vec{B}|$
(C) $\vec{C}$ must be greater than $|\vec{A}-\vec{B}|$ and
(D) $\vec{C}$ may be equal to $|\vec{A}-\vec{C}|$.

Answer (C)