SAK

ACADEMY

XI, XII, NEET/AIIMS

IIT-JEE(MAIN+ADV.)

TOPIC:-

(01)PHYSICAL QUANTITY

(02)VECTOR NOTATION

(03)PROPERTIES OF VECTOR

(04)ANGEL BETWEEN TWO VECTORS

(05)TYPES OF VECTORS

(06)BASIC NUMERICAL

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decreasing.

Thus, we can say that number is inversely proportional to unit.

i.e. $\boxed{n \propto \frac{1}{u} \text{ or } u \propto \frac{1}{n} \text{ or } nu = K}$

We can also write $\boxed{n_1 u_1 = n_2 u_2 = n_3 u_3 = \cdots = K}$

Physical quantities can be classified into two classes on the basis of directions:-

<i> Scalar physical quantity & <ii> Vector physical quantity.

<i> Scalars or scalar physical quantities → Those physical quantities which have magnitude only and do not need directions to represent are called scalars.

OR

Scalars → Those physical quantities which are described completely by its magnitude only and does not require a direction are called scalar quantities.

For Example:- Mass, Time, Volume, Density, Work etc. are few examples of scalars quantities.

Note:- Operation on scalar quantities such as addition, subtraction, multiplication or division can be done according to the general rules of algebra.

Vector quantities or vector physical quantities → Those physical quantities which have both magnitude as well as direction and obey the law of vector addition or vector algebra (i.e. the triangle and the parallelogram law of vector addition are called vector quantities.

For Example:- Displacement, velocity, acceleration, force, torque, etc. are few examples of vectors.

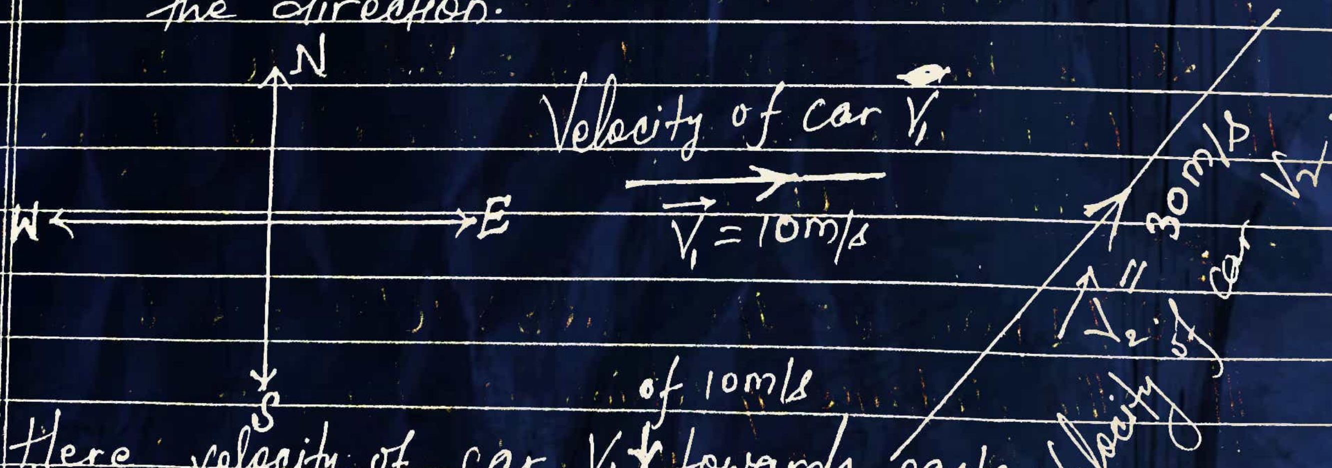
Note:- The physical quantity current has both magnitude and direction but it is still a scalar as it disobeys the laws of vector algebra.

Vector notation :→ Usually a vector is represented by a bold capital letter with an arrow over its head or without arrow as $\vec{A}$, $\vec{B}$, $\vec{C}$ or simply **A, B, C**.

The magnitude of a vector $\vec{A}$ or **A** is represented by $|\vec{A}|$ or A.

Modulus of a vector :→ The magnitude of a vector is known as modulus of a vector.

Graphical representation of a vector :→ A vector is graphically represented by an arrow drawn to a chosen scale, parallel to the direction of the vector. The length of the arrow represents the magnitude and the tip of the arrow (arrow-head) represents the direction.

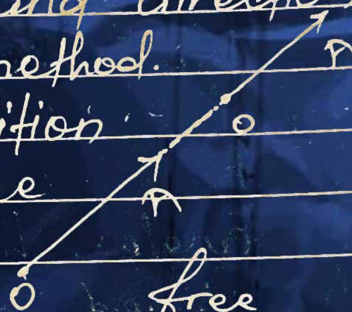


Here velocity of car V_1 of 10m/s towards east; and another car V_2 moving with a speed of 30 m/s towards north-east. It is known as G.R.

PROPERTIES OF VECTORS :-

<01> **Vectors can be bound :-** The vectors in which point of application and direction both are fixed. such type of vector's position is fixed and it is called bound vectors.
For example.  $\vec{OA}$ is a bound vector.

<02> **Vectors can be sliding or shifting :-** In sliding the point of application is shifted along the original line of action without any change in magnitude and direction.
It is also called shifting method.
For example :- Shifted the position of $\vec{OA}$ is shown in the given figure as shown

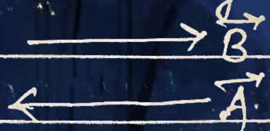


<03> **Vectors can be moved freely :-** In free movement of vectors, its point of application can be changed without any change in magnitude and direction of the vectors and are always parallel to the original line of action
It is also known as parallel shifting method.
For example : As $\vec{OA}$ has been shifted in a new position as shown in Fig. given :

<04> **Angle between collinear vectors is always zero or 180°**

<(i)> 

$\theta = 0°$

<(ii)> 

$\theta = 180°$

<05> **If a vector is displaced parallel to itself, then it does not change.**
For example  Here $\vec{A}$ and $\vec{B}$ are same. i.e $\vec{A} = \vec{B}$

⟨6⟩ If a vector is rotated by an angle ($\theta \neq 2n\pi$, where $n = 1, 2, 3, 4, 5, \ldots$) then the vector is changed but when a vector is rotated by an angle ($\theta = 2n\pi$, where $n = 0, 1, 2, 3, 4, 5 \ldots$ etc), then the vector does not change.

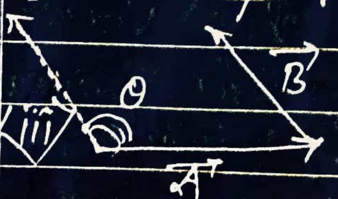
ANGLES BETWEEN TWO VECTORS

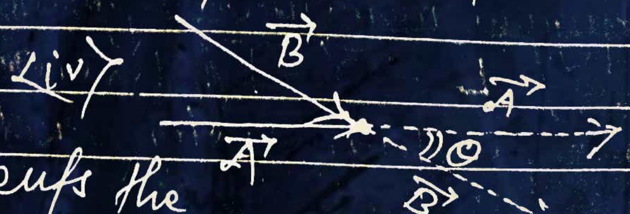
Angle between two vectors :- The shortest angle between two vectors of which any of the two vectors is rotated about the other vector such that both of the vectors have the same direction is known as angle between two vectors. OR.

The shortest angle between two vectors is said to be angle between two vectors if it is formed by joining these two vectors by tail to tail using shifting method. OR

The smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself is known as the angle between the two vectors.

For example :-





Here θ represents the angle between $\vec{A}$ and $\vec{B}$.

Note:- The angle between ($\vec{A}$ and $\vec{B}$) ~~two~~ vectors is always greater than or equal to zero (0°) degree and less than or equal to 180°. (i.e. $0° \leq \theta \leq 180°$).

TYPES OF VECTORS

<01> Polar vectors:- Those vectors which have a starting point or a point of application are known as polar vectors.
For example:- Displacement, force, velocity---etc.

<02> Axial vectors:- Those vectors which represent rotational effect and act along the axis of rotation in accordance with right hand thumb/screw rule are known as axial vectors.
Example:- Angular velocity, angular acceleration, torque, angular momentum--etc.

Some important facts about axial vectors

<i> Relation between linear velocity and angular velocity. i.e. $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ wher $\vec{v}$ = linear velocity, $\vec{\omega}$ = angular velocity and $\vec{r}$ = radius, here $\vec{\omega}$ read as omega.

<ii> $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$ Where $\vec{\tau}$ = Torque, $\vec{r}$ = position vector & $\vec{F}$ = normal force or $\vec{F}$ = perpendicular force & τ is read as tau.

<iii> $\boxed{\vec{L} = \vec{r} \times \vec{p}}$ Where $\vec{L}$ = Angular momentum, $\vec{r}$ = position vector and $\vec{p}$ = linear momentum.

<iv> $\boxed{W = \vec{F} \cdot \vec{S}}$ Where W = work done, $\vec{F}$ = force and $\vec{s}$ = displacement.

(03) Equal vectors :→ Those vectors which have equal magnitude and same direction are called equal vectors.

For Example: $\xrightarrow{\vec{A}}$ vectors $\vec{A}$ and $\vec{B}$ are
$\xrightarrow{\vec{B}}$ equal vectors. i.e $\boxed{\vec{A} = \vec{B}}$

Note:- The angle between equal vectors is zero (0°).

(04) Parallel vectors :→ Those vectors which have same direction and magnitude may be equal or different are called parallel vectors.

For Example: $\xrightarrow{\vec{A}}$ Vectors $\vec{A}$ and $\vec{B}$
$\xrightarrow{\quad\vec{B}\quad}$ are parallel.

Note:- Two equal vectors are always parallel but, two parallel vectors may not be equal vectors.

(05) Anti-parallel vectors :→ Those vectors which have opposite direction and magnitude may be equal or different are called anti-parallel vectors.

For Example:- $\xrightarrow{\vec{A}}$ Vectors $\vec{A}$ and $\vec{B}$ are
$\xleftarrow{-\vec{B}}$ anti-parallel vectors.

(06) Negative vectors :→ Those vectors of a given vectors which have same magnitude but acting in a direction opposite to that of the given vector are called negative vectors.

For Example $\xrightarrow{\vec{A}}$ Here Vector $\vec{B}$ is the
$\xleftarrow{\vec{B}}$ negative vector of $\vec{A}$

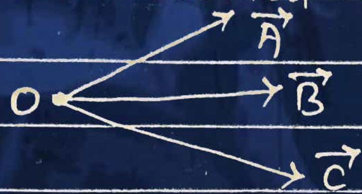
The It can be written as $\boxed{\vec{A} = -\vec{B}}$

Note:- (i) The angle between negative vectors is π rad or 180°.

(ii) The direction of a negative vectors is always opposite to the given vector.

(07) Co-initial vectors (Concurrent vectors) :→ Those vectors which have the same / Common initial points are called co-initial vectors.

For Example:-



Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ have common initial points. So, They are concurrent vectors.

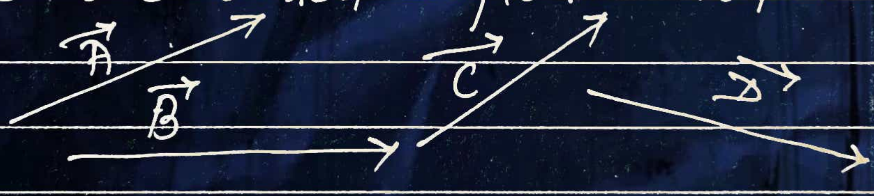
(08) Collinear vectors :→ Those vectors which have equal or unequal magnitude and act along same line are called collinear vectors.

For Example:-

Note:- The angle between collinear vectors can be zero or 180°.

(09) Coplanar vectors :→ Those vectors which lie in the same plane and act in the same plane are called coplanar vectors.

For Example:-



The ~~four~~ vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ are acting in the plane of paper.

(10) Localised vectors :→ Those vectors whose initial point is fixed are called localised vectors.

Note:- It is also known as fixed vectors.

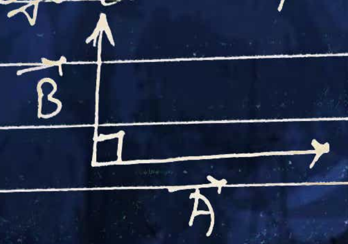
(11) Non-localised vectors :→ Those vectors whose initial point is not fixed are called non-localised vectors.

Note:- It is ~~called~~ also called a free vector.

(12) **Orthogonal vectors** :→ Two vectors are said to be orthogonal if the angle between them is 90°. OR

Those two vectors which are perpendicular to each-other are called orthogonal vectors.

For Example:-



The vectors $\vec{A}$ and $\vec{B}$ are orthogonal to one another.

(13) **Zero vector or Null vector** :→ A vector having zero magnitude is called a null vector or zero vector

Note:- (i) Zero vector has no specific direction.
(ii) The position vector of origin is zero vector.
(iii) Zero vectors are only of mathematical importance
(iv) $0 \times \vec{A} = \vec{O}$ = Zero vector.

(14) **Unit vector** :→ A vector whose magnitude is unit and points in a particular directions, is called unit vector.

Note:- (i) It is represented by $\hat{A}$ and read as A cap or A hat or A caret.
(ii) The unit vector along the direction of $\vec{A}$ is

$$\boxed{\hat{A} = \frac{\text{Vector}}{\text{magnitude of the vector}}} \quad \text{OR}$$

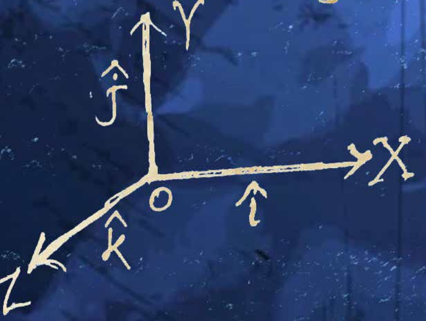
$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}} \quad \text{OR} \quad \boxed{\vec{A} = \hat{A}|\vec{A}|}$$

(iii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$,

then $\boxed{|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}}$

(i) In cartesian coordinate, The unit vectors along x-axis, y-axis and z-axis are $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

$$\boxed{|\hat{i}| = |\hat{j}| = |\hat{k}| = 1}$$



(ii) A unit vector is used to specify the direction of a vector.

(iii) The unit vector is unitless and dimensionless vector and represents direction only

(iv) Condition for two vectors for being parallel vectors; Anti-parallel vectors or collinear vectors

(i) Two vectors $\vec{A}$ and $\vec{B}$ will be parallel if and only if $\vec{A} = \lambda\vec{B}$ OR $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = \lambda$

Proof Let two vectors are $\vec{A}$ and $\vec{B}$ where $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

We know that, if two vectors are parall then their direction (i.e. unit vector) must be same.

Hence, $\hat{A} = \hat{B} \Rightarrow \frac{\vec{A}}{A} = \frac{\vec{B}}{B}$

$\Rightarrow \vec{A} = \left(\frac{A}{B}\right)\vec{B} \Rightarrow \vec{A} = \lambda\vec{B}$ where $\lambda = \frac{A}{B}$ = positive constant

Thus $\boxed{\text{Two vectors are parallel if } \vec{A} = \lambda\vec{B}}$

Already we have proved that $\vec{A} = \lambda \vec{B}$

Hence, $A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = \lambda(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\Rightarrow A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = \lambda B_x\hat{i} + \lambda B_y\hat{j} + \lambda B_z\hat{k}$

On comparing each term of both sides we get

$A_x = \lambda B_x$, $A_y = \lambda B_y$ and $A_z = \lambda B_z$

$\Rightarrow \lambda = \frac{A_x}{B_x}$, $\lambda = \frac{A_y}{B_y}$ and $\lambda = \frac{A_z}{B_z}$

Hence, $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = \lambda$ = positive constant $= \frac{A}{B}$

Thus, Two vectors $\vec{A}$ and $\vec{B}$ are parallel if

$$\boxed{\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = \lambda = \frac{A}{B} = \text{positive constant}}$$

(IX) Two vectors are anti-parallel if $\vec{A} = -\lambda\vec{B}$ OR

$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = -\lambda = \frac{-A}{B}$ = negative constant

(X) Two vectors $\vec{A}$ and $\vec{B}$ such that $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are collinear if

$$\boxed{\begin{gathered}\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = \lambda = \frac{A}{B} = \text{positive constant OR} \\ \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = -\lambda = \frac{-A}{B} = \text{Negative constant}\end{gathered}}$$

MULTIPLICATION AND DIVISION OF VECTORS BY SCALARS

Multiplication :→ The product of a vector $\vec{A}$ and a scalar m is a vector $m\vec{A}$ whose magnitude is m times the magnitude of $\vec{A}$ and which is in the direction or opposite to $\vec{A}$ according as the scalar m is positive or negative. Thus, $\boxed{m(\vec{A}) = m\vec{A}}$

Further, if m and n are two scalars, then

$\boxed{(m+n)\vec{A} = m\vec{A} + n\vec{A}}$ OR $\boxed{n(m\vec{A}) = (mn)\vec{A}}$

Division :→ The division of vector $\vec{A}$ by a non-zero scalar m is defined as the multiplication of $\vec{A}$ by $\frac{1}{m}$. i.e. $\boxed{\frac{\vec{A}}{m} = \vec{A} \times \frac{1}{m}}$

QUESTIONS BASED ON ABOVE FACTS

SET-A

Q(01) Two vectors $\vec{A}$ and $\vec{B}$ are given such that $\vec{A} = 2\hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{B} = 14\hat{i} + 21\hat{j} - 49\hat{k}$, find which one of the following is correct

(A) $\vec{A}$ and $\vec{B}$ are parallel (B) $\vec{A}$ and $\vec{B}$ are anti-parallel

(C) $\vec{A}$ and $\vec{B}$ are not parallel (C) All of these

Q.(2) If two vectors $\vec{A}$ and $\vec{B}$ are given such that $\vec{A} = 2\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{B} = \alpha\hat{i} + 6\hat{j} + \beta\hat{k}$ then find the value of α and β are if $\vec{A}$ and $\vec{B}$ are parallel

<A> 2, 3 <B> 2, −3 <C> 2, 6 <D> 6, −3

Q.(03) The position vectors of four points A, B, C and D are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 7\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{d} = 3\hat{i} + 6\hat{j} + 9\hat{k}$ respectively. Examine whether vectors $\vec{AB}$ and $\vec{CD}$ are Collinear.

<A> Collinear <B> parallel <C> (A) and (B) both (D) None of these.

SET-2

Q.(01) Which is not a vector quantity?

<A> Current <B> Displacement <C> velocity <D> Acceleration

Q <02> Which one of the following is a scalar quantity?

<A> Dipole moment <B> Electric field <C> Acceleration <D> Work

Q.03. Which one of the following is not the vector quantity?
<A> Torque <B> Displacement <C> Velocity <D> Speed

Q.04. Which one is a vector quantity?
<A> Time <B> Temperature <C> Magnetic flux
<D> Magnetic field intensity

Q.<05> Which one of the following statement is false?
<A> A vector cannot be displaced from one point to another point.
<B> Distance is a scalar quantity but displacement is a vector quantity.
<C> Momentum, force and torque are vector quantities.
<D> Mass, speed and energy are scalar quantities.

Q.06. Pressure is
<A> Scalar <B> Vector <C> Both (A) & (B) <D> None of these

Q.07. Which of the following is not the vector quantity?
<A> Torque <B> Displacement <C> Dipole moment
<D> Electric flux

Q.08. Which one of the following is a vector?
<A> Pressure <B> Surface tension <C> Moment of inertia
<D> None of these

Q.09. Surface area is
<A> scalar <B> Vector <C> Neither scalar or nor vector
<D> Both (A) and <B>

Q.10. Which is a vector quantity?
<A> Angular momentum ~~<B> Momentum~~ <B> Work
<C> Potential energy <D> Electric current

Q. 11. Which one is a vector quantity?
<A> Temperature <B> Momentum <C> Work <D> Speed

Q. 12. Which is a vector quantity?
<A> Work <B> Power <C> Torque <D> Gravitational Constant.

Q. 13. A vector is not changed if
<A> it is rotated through an arbitrary angle
<B> it is multiplied by an arbitrary scalar
<C> it is cross multiplied by a unit vector
<D> it is displaced parallel to itself

Q. 14. The forces, which meet at one point but their lines of action do not lie in one plane, are called
<A> non-coplanar non-concurrent forces
<B> non-coplanar concurrent forces
<C> coplanar concurrent forces
<D> Coplanar non-concurrent forces

Q. 15. Unit vector does not have any specified
<A> direction <B> magnitude <C> unit <D> All of these

Q. 16. The magnitude of $(\hat{i}+\hat{j})$ is
<A> 2 <B> 0 <C> $\sqrt{2}$ <D> 4

Q. 17. The unit vector along $(\hat{i}+\hat{j})$ is
<A> $\hat{k}$ <B> $\hat{i}+\hat{j}$ <C> $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ <D> $\frac{\hat{i}+\hat{j}}{2}$

Q. 18. What happens, when we multiply a vector by (−2)?
<A> direction reverses and unit changes
<B> direction reverses and magnitude is doubled.
<C> direction remains unchanged and unit changes
<D> None of the above

Q. 19. Which of the following represents a unit vector?
<A> $\frac{|\vec{A}|}{\vec{A}}$ <B> $\frac{\vec{A}}{|\vec{A}|}$ <C> $\frac{|\vec{A}|}{|\vec{A}|}$ <D> $\frac{\vec{A}}{\vec{A}}$

Q.20. A vector multiplied by the number 0, results into <A> 0 <B> $\vec{A}$ <C> $\vec{0}$ <D> $\hat{A}$.

Q.21. Find the vector that must be added to the vector $(\hat{i}-3\hat{j}+2\hat{k})$ and $(3\hat{i}+6\hat{j}-7\hat{k})$ so that the resultant vector is a unit vector along the y-axis.
<A> $-4\hat{i}-2\hat{j}+5\hat{k}$ <B> $-4\hat{i}+2\hat{j}+5\hat{k}$ <C> $4\hat{i}-2\hat{j}+5\hat{k}$
<D> $-4\hat{i}-2\hat{j}-5\hat{k}$

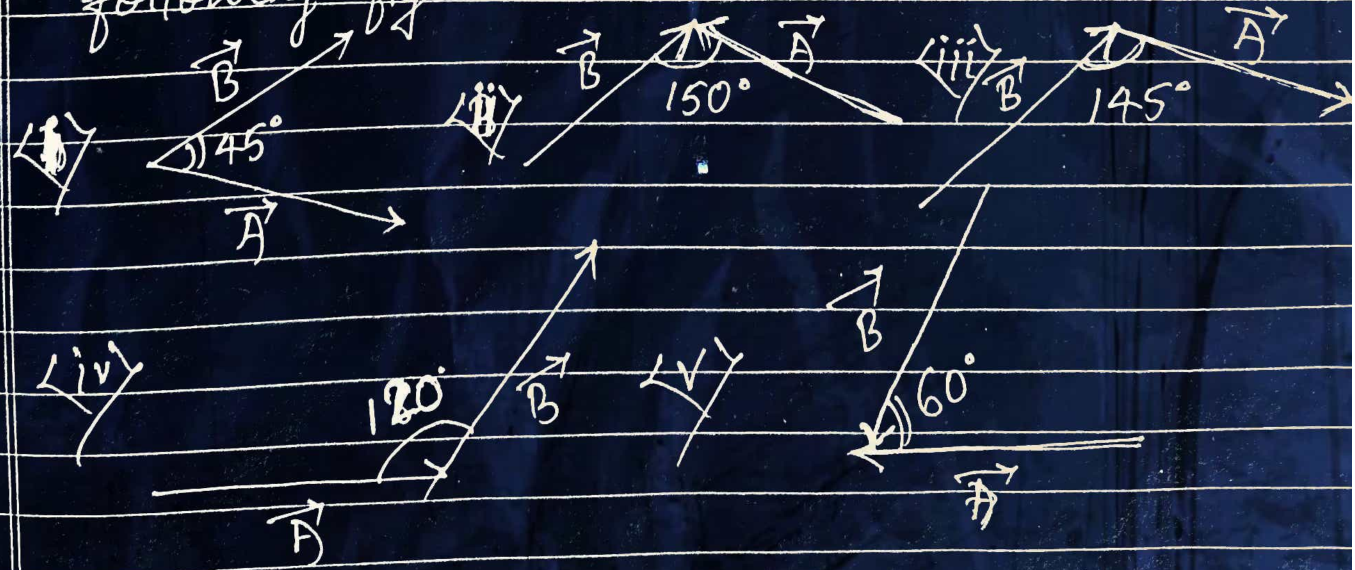
Q.22. Check whether the vector $\left(\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}\right)$ is a unit vector or not.

Q.23. Find the unit vector of $(4\hat{i}-3\hat{j}+\hat{k})$.
<A> $\sqrt{26}$ <B> $\frac{4\hat{i}-3\hat{j}+\hat{k}}{\sqrt{26}}$ <C> $\sqrt{26}(4\hat{i}-3\hat{j}+\hat{k})$ <D> None of these

Q.24. If vectors $(\hat{i}-3\hat{j}+5\hat{k})$ and $(\hat{i}-3\hat{j}-a\hat{k})$ are equal vectors, then find the value of a.
<A> 5 <B> -5 <C> ± 5 <D> All of these

Q.25. If $\vec{A}=4\hat{i}+3\hat{j}$ and $\vec{B}=24\hat{i}+7\hat{j}$, find the vector having the same magnitude as B and parallel to A.
<A> $(\hat{i}+\hat{j})$ <B> $4\hat{i}+3\hat{j}$ <C> $20\hat{i}+15\hat{j}$ <D> $15\hat{i}+20\hat{j}$

Q.26. Find the angle between $\vec{A}$ and $\vec{B}$ from the following figures:



Q.<27> What is the angle between $\vec{a}$ and $-3/2\,\vec{a}$?

<A> 90° <B> 0° <C> 180° <D> 360°

Q.<28> What is the angle between $2\vec{a}$ and $4\vec{a}$?

<A> 0° <B> 90° <C> 180° <D> None of these

Q.<29> What is the angle between $3\vec{a}$ and $-5\vec{a}$? what is the ratio of magnitude of two vectors?

<A> 0°, 3 <B> 90°, −5 <C> 180°, 0·6 <D> None of these

Q.<30> Obtained the magnitude of $2\vec{A}-3\vec{B}$, if $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+\hat{k}$.

<A> $\sqrt{6}$ <B> $-\sqrt{6}$ <C> $\sqrt{90}$ <D> None of these

SET-B

ANSWER KEY:- 01.(A), 02.(D), 03.(D), 04.(D)
05.(A), 06.(A), 07.(D), 08.(D), 09.(A), 10.(A),
11.(B), 12.(C), 13.(D), 14.(B), 15.(C), 16.(C), 17.(C),
18.(B), 19.(B), 20.(C), 21.(A), 22. Yes, it is a unit vector.
23.<B>, 24.(B), 25.(C), 26.(i) 45° <ii> 150° <iii> 35 <iv> 60°
<v> 60°, 27.(C), 28.(A), 29.(C), 30.(C)

SET-A, ANSWER KEY

01 (A), 2.(B), 3.