SAK ACADEMY

XI XII NEET AIIMS IIT-JEE (MAIN+ADV.)

TOPICS:-

1.SPEED

2.VELOCITY

3.IMPORTANT FACTS ABOUT SPEED & VELOCITY

4.CALCULUS APPROACH TO SPEED & 5.VELOCITY

6.SOME QUESTIONS BASED ON ABOVE FACTS

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SPEED & VELOCITY

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Speed :- The distance travelled by a body per unit time is called speed of the body.

If a body covers 80km distance in a hour then we can say that the speed of the body is 80km/hr.

Mathematical calculation of speed.

Let distance travelled by the body is S. and time taken to travell S distance is t.

$\therefore$ Distance travelled in ts is S

$\therefore$ " " " 1s " $\frac{S}{t}$

We known that distance travell in unit time is speed.

Hence $\boxed{V = S/t}$

Where V = speed, S = distance & t = time.

Some important fact about speed

(i) Speed is a scalar quantity.

(ii) It is generally denoted by symbol V.

(iii) The SI unit of speed is m/s or ms^{-1} but its C.G.S unit is cm/s or cms^{-1}.

(iv) The Dimensional formula is $[M^0LT^{-1}]$ OR $[V] = [M^0LT^{-1}]$.

(v) The speed of a body can be 0 or positive but it cannot be negative.

(vi) The speed is always greater than or equal to velocity. i.e. $|v| \geq |\vec{v}|$

TYPES OF SPEED

- Uniform speed OR Constant speed
- Non-uniform speed OR Variable speed
- Average speed
- Instantaneous speed

(A) Uniform speed :- If a particle covers equal distances in equal intervals of time, (where no matter how small the interval are) then it is said to be moving with uniform speed.

(B) Non-uniform speed :- A body is said to be non-uniform speed if it covers unequal distance in equal interval of time.

(C) Average speed :- The ratio of the total distance travelled by a particle to the total time taken by the particle for a given interval of time is known as average speed.

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Average speed = Distance travelled / Time taken OR

$$V_{av} = \frac{\Delta S}{\Delta t}$$ OR $$V_{av} = \frac{S_f - S_i}{t_2 - t_1}$$

Time average speed:- When a particle moves with different uniform speed V_1, V_2, V_3 ---- etc in different time intervals t_1, t_2, t_3, ---- etc. respectively, its average speed over the total time of journey is given as.

$$V_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$V_{av} = \frac{S_1 + S_2 + S_3 + \cdots + S_n}{t_1 + t_2 + t_3 + \cdots + t_n}$$

$$V_{av} = \frac{V_1 t_1 + V_2 t_2 + V_3 t_3 + \cdots + V_n t_n}{t_1 + t_2 + t_3 + \cdots + t_n}$$

SPECIAL CASES :-

Case-I When a particle moves with speed 'u' upto half time of its total motion and rest time it is moving with speed V

then $$V_{av} = \frac{u+v}{2}$$

Explanation :- $$V_{av} = \frac{S_1 + S_2}{\frac{t}{2} + \frac{t}{2}} = \frac{u\frac{t}{2} + v\frac{t}{2}}{2t}$$

$$V_{av} = \frac{ut + vt}{2} \times \frac{1}{t} = \frac{t(u+v)}{2t}$$

$$V_{av} = \frac{u+v}{2}$$

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Distance average speed:- When a particle describes different distances $s_1, s_2, s_3, \ldots$ with different time intervals $t_1, t_2, t_3, \ldots$ with speeds $v_1, v_2, v_3 \ldots$ respectively then the speed of particle average over the total distance can be given as

$$V_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$V_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

$$\boxed{V_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots}}$$

When a particle moves the first half of a distance at a speed of v_1 and second half of the distance of the speed v_2

then $\boxed{V_{av} = \frac{2v_1 v_2}{v_1 + v_2}}$

Explanation:- $V_{av} = \frac{\text{Total distance covered}}{\text{total time taken}}$

$$V_{av} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{\frac{s}{2} + \frac{s}{2}}{\frac{\frac{s}{2}}{v_1} + \frac{\frac{s}{2}}{v_2}}$$

$$V_{av} = \frac{\frac{2s}{2}}{\frac{s}{2v_1} + \frac{s}{2v_2}} = \frac{s}{\frac{v_2 + v_1}{2v_1 v_2}} = s \times \frac{2v_1 v_2}{s(v_1 + v_2)}$$

$$\boxed{V_{av} = \frac{2v_1 v_2}{v_1 + v_2}}$$ proved

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When a particle covers one-third distance at speed v_1, next one third at a speed v_2 and last one third at speed v_3, then $\boxed{v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}}$

Instantaneous speed :- The speed of a particle at any instant is known as instantaneous speed. OR The rate of change of speed with respect to time is known as instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e. $\Delta t \to 0$). Thus the instantaneous speed $\boxed{v_{ins} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}} \Rightarrow \boxed{v_{ins} = \frac{ds}{dt}}$

Calculus approach of distance & speed.

$\therefore \boxed{v = \frac{ds}{dt}}$ Note :- Differentiation of distance w.r.t time is speed.

Hence, $\boxed{s = \int_{t_1}^{t_2} v\,dt}$ Note :- Integration of v w.r.t time given in a given interval of time gives distance.

Q① The distance travelled by a particle moving in a straight line is given by

$s = 2t^2 + t + 1$ where s is in meter and t is in second. The average speed and instantaneous speed of the particle in 2 second is

(A) 5 m/s, 10 m/s (B) 5 m/s, 9 m/s (C) All of the

Sol^n:- when $t_1 = 0$ sec then $s = 1$ m but

when $t_2 = 2$ sec then $s = 11$ m

$$\therefore \text{Vav} = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_2 - t_1} = \frac{11-1}{2-0} = \frac{10}{2} = 5 \text{ m/s}$$

Now, $v = \frac{ds}{dt} = \frac{d}{dt}(2t^2 + t + 1) = 4t + 1$

Hence, v at $t = 2$ s $= 4 \times 2 + 1 = 9$ m/s

Hence, correct option is (B).

Q(2) A body starts from rest and its speed is the function of time given by $v = 3t^2 - 2t + 5$ then the distance travelled by the body in 2 second is

(A) 10 m (B) 12 m (C) 14 m (D) 16 m

Sol^n:- We know that

$$s = \int_{t_1}^{t_2} v\,dt \Rightarrow s = \int_0^2 (3t^2 - 2t + 5)\,dt$$

$$s = \left[t^3 - t^2 + 5t\right]_0^2 \Rightarrow s = 2^3 - 2^2 + 5 \times 2$$

$$s = 8 - 4 + 10 = 14 \text{ m}$$

Hence correct option is (C)

VELOCITY

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Velocity :- The displacement covered by a particle in per unit time is known as velocity.

Calculation of velocity

Mathematical calculation of magnitude of velocity.

∵ If magnitude of displacement of a particle in time t is $|\vec{s}|$.

∴ The magnitude of displacement of a particle in 1's is $\frac{|\vec{s}|}{t}$ and

We know that displacement covered in unit time is velocity

Hence $$|\vec{v}| = \frac{|\vec{s}|}{t}$$ magnitude of velocity = $|\vec{v}|$ and $|\vec{s}|$ = magnitude of displacement and t = time.

Now, $$\vec{v} = \frac{\vec{s}}{t}$$ where $\vec{v}$ = velocity
$\vec{s}$ = Displacement &
t = time taken

Some important facts about velocity.

(i) The S.I unit of velocity is same as speed i.e. m/s or ms^{-1} and its C.G.S unit is cm/s or cms^{-1}.

(ii) It is a vector quantity.

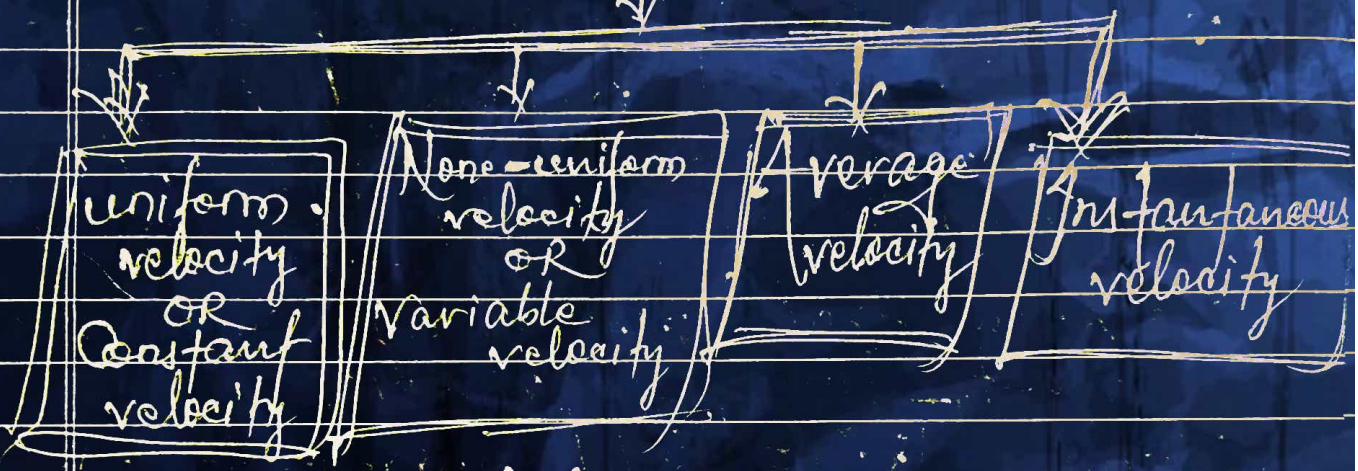
(iii) It is generally denoted by $\vec{v}$.

(iv) The dimension of velocity is same as speed i.e. $[\vec{v}] = [M^0 L T^{-1}]$

(v) It is always less than or equal to speed.

(vi) velocity can be zero (0), positive (+ve) or negative (-ve).

TYPES OF VELOCITY



(A) **Uniform velocity** → A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and that is possible only when the particle moves in same straight line without reversing its direction.

(B) **Non-uniform velocity** → A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes or both changes.

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(c) Average velocity :- It is defined as the ratio of displacement to time taken by the body.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{time taken}}$$

$$V_{av} = \frac{\Delta \vec{r}}{\Delta t} \Rightarrow V_{av} = \frac{\vec{r}_f - \vec{r}_i}{t_2 - t_1}$$

(d) Instantaneous velocity :- The rate of change of position vector of particles with time at a certain instant of time is known as instantaneous velocity.

Instantaneous velocity $\vec{V}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$

OR $\vec{V}_{ins} = \frac{d\vec{r}}{dt}$

(e) Calculas approach of displacement & velocity.

$$\therefore \vec{V} = \frac{d\vec{r}}{dt} \Rightarrow \vec{V}\,dt = d\vec{r}$$

$$\int_{r_i}^{r_f} d\vec{r} = \int_{t_i}^{t_f} \vec{V}\,dt \Rightarrow [\vec{r}]_{r_i}^{r_f} = \int_{t_i}^{t_f} \vec{V}\,dt$$

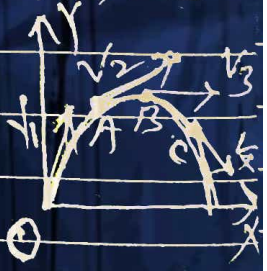
$$\vec{r}_f - \vec{r}_i = \int_{t_i}^{t_f} \vec{V}\,dt$$

Hence, $\vec{r} = \int_{t_i}^{t_f} \vec{v}\,dt$ OR $\vec{r}_f - \vec{r}_i = \int_{t_i}^{t_f} \vec{v}\,dt$

Comparison between instantaneous speed and instantaneous velocity.

(A) Instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is V_1, at point A the instantaneous velocity of stone is V_2, similarly at point B and C are V_3 and V_4 respectively



Note:- Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(B) A particle may have constant instantaneous speed but variable instantaneous velocity.

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Example - When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(2) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(3) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(4) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

then Instantaneous velocity, $\vec{v} = \frac{d\vec{x}}{dt}$

$$\vec{v} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$$

$$\vec{v} = -A_1 + 2A_2 t$$

For the given value of t, we can find out the instantaneous velocity.

eg: for $t=0$, Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

Comparison between average speed and average velocity

(A) $|\text{Average velocity}| / |\text{Average speed}| \leq 1$.

$\Rightarrow$ $|\text{Average speed}| \geq |\text{Average velocity}|$

(B) Average speed is scalar while average velocity is a vector.

(C) Both average speed and average velocity having same units m/s or $m.s^{-1}$.

(D) Both average speed and average velocity have same dimensions $[L.T^{-1}]$.

(E) Average speed or velocity depends on time interval over which it is defined.

(F) For a given time interval average velocity is single valued ~~which~~ while average speed can have many values depending on path followed.

(G) If after motion body comes back to its initial position then $\vec{V}_{av} = \vec{0}$ as $\Delta\vec{r} = 0$ but $V_{av} > 0$ and finite as $\Delta s > 0$.

(H) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be i.e. $V_{av} > 0$ while $\vec{V}_{av} = 0$ or $\vec{V} < 0$

i.e. Average velocity could be zero or positive or negative but average speed is always positive for a moving body.

(I) If a particle travels distances S_1, S_2, S_3, ... etc. with speeds V_1, V_2

v_3, ---------- etc., in same direction, then the distance travelled $= s_1 + s_2 + s_3 + \text{----------}$

$$\text{Total distance travelled} = s_1 + s_2 + s_3 + \cdots + s_n$$

$$\text{Total time taken} = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \text{----------}$$

$$\text{Average speed} = v_{av} = \frac{s_1 + s_2 + s_3 + \text{----------}}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \text{----------}}$$

If $s_1 = s_2 = s$, i.e. the body covers equal distance with different speeds then

$$v_{av} = \frac{2s}{\left(\frac{1}{v_1} + \frac{1}{v_2}\right)s} \Rightarrow v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

(J) If a particle travels with speed v_1, v_2, v_3, v_4 ----, ----, etc., during time intervals t_1, t_2, t_3, ----------, etc., then total distance travelled, $s = v_1t_1 + v_2t_2 + v_3t_3 + \text{----------}$

$$\text{Total time taken} = t_1 + t_2 + t_3 + \text{----------}$$

So, Average speed, $v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \cdots\cdots}{t_1 + t_2 + t_3 + \text{----------}}$

(K) If $t_1 = t_2 = t_3 = \text{----------} = t_n$, the

$$v_{av} = \frac{(v_1 + v_2 + v_3 + \cdots\cdots + v_n)t}{nt}$$

$$v_{av} = \frac{v_1 + v_2 + v_3 + \text{----------} + v_n}{n}$$

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Q(01) A body of mass m is moving with a uniform velocity $\vec{v}$. Find the change in velocity after (A) Completing quater revolution (B) traveling an arc of 60°
(i) $v, \sqrt{2}v$ (ii) v, v (iii) $\sqrt{2}v, v$ (iv) $\frac{\sqrt{3}}{2}v, v$

Ans: (iii)

Q(02) If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is [MP PMT-2003]
(A) $\frac{1}{2}\sqrt{v_1 v_2}$ (B) $\frac{v_1+v_2}{2}$ (C) $\frac{2v_1v_2}{v_1+v_2}$ (D) $\frac{5v_1v_2}{3v_1+2v_2}$

Ans: (D)

Q(03) A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min. is equal to Ans: (D)
(A) 5 km/h (B) $\frac{25}{4}$ km/h (C) $\frac{30}{4}$ km/h (D) $\frac{45}{8}$ km/h

Q(04) The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in metres and t in second. The displacement, when velocity is zero, is
(A) 24 meter (B) 12 m (C) 5 m (D) zero Ans: (D)

Q(05) The motion of a particle is described by the equation $x = a + bt^2$ where $a = 15$ cm and $b = 3$ cm. It's instantaneous velocity at

time 3 second will be [AMU (Med.) - 2000] Ans: (B)

(A) 36 cm/sec (B) 18 cm/sec (C) 16 cm/sec (D) 32 cm/sec

(06) A train has a speed of 60 km/h for the first one hour and 40 km/h for the next half hour. Its average speed in km/h is [JIPMER-199] Ans: (B)

(A) 50 (B) 53.33 (C) 48 (D) 70

(07) A person completes half of its his journey with speed V_1 and rest half with speed V_2. The average speed of the person is [RPET-1993, MP PMT-2001]

(A) $V_{av} = \frac{V_1 + V_2}{2}$ (B) $V_{av} = \frac{2V_1V_2}{V_1+V_2}$ Ans: (B) (C) $V_{av} = \frac{V_1V_2}{V_1+V_2}$ (D) $V_{av} = \sqrt{V_1V_2}$

(08) A car moving on a straight road covers one third of the distance with 20 km/hr. and the rest with 60 km/hr. The average speed is Ans: (D) [MP PMT-1999, CPMT-2002]

(A) 40 km/hr. (B) 80 km/h (C) $46\frac{2}{3}$ km/hr (D) 36 km/h.